High-Power Microwave Rejection Filters Using Higher-Order Modes*

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Summary—In order to obtain filters capable of handling very high power, the use of radial lines and uniform line discontinuities was investigated. Forty-five-degree tapers and uniform lines were used to design a high-power microwave filter capable of handling 700 kw at 15 pounds pressure in a 0.900 by 0.400 ID waveguide. In addition to the filtering which results from the discontinuities in the TE_{10} mode in the waveguide, high insertion loss elements are effected when the enlarged uniform line section is larger than the TE_{10} mode waveguide wavelength and when the length of the enlarged section is approximately $(2n-1)\lambda_g/4$. Extremely large insertion losses are possible by the cascading of these elements. Tuning, in the standard-size waveguide, has no effect on the insertion loss of the higher-mode enlarged waveguide at its resonant frequency. Empirical design formulas are evolved and the design procedure for band-rejection filters is given, using these high insertion loss elements.

Introduction

As the power of radar and communications transmitters increase and as their number continually becomes larger, the mutual interference between radars at the same site, or between radars and communications, reaches critical proportions. To eliminate the large power at spurious frequencies which are far removed from the assigned operating frequency band, it is essential to provide adequate filtering at high power on the transmission lines between the transmitters and the antennas. As an outgrowth of work done on high-power filters using radial transmission lines in combination with enlarged uniform transmission lines, it was found that the higher-order mode in the enlarged line would provide a useful high insertion loss element, which in combination with the filtering effect of the discontinuities in the fundamental TE_{10} mode, would provide a solution to the high-power microwave filtering problem. The basic filter section consists of a waveguide of rectangular cross section whose narrow dimension is expanded through a taper and then contracted through another taper to its original dimensions. The two tapers are separated by a length of an enlarged uniform line. The basic filter section (see Fig. 1) and its reactive characteristics have been described in a previous paper.[1] In this paper, this technique is extended to the case where higher-order modes can propagate in the enlarged uniform line in the filter section.

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[1] J. H. Vogelman, "High-power microwave filters," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 429–439; October, 1958.

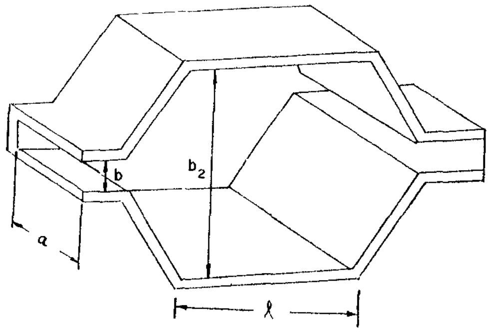


Fig. 1—Typical filter section with one side wall removed.

The High Insertion Loss Section

In filter sections (see Fig. 1) with enlarged center sections of a size where the height is greater than the waveguide wavelength in the standard TE_{10} mode waveguide, frequencies can be found where the insertion loss rises sharply to values in excess of 35 to 45 db for a single section. This has been experimentally investigated in an attempt to specify the conditions for its occurrence, in order to permit its use in the design of rejection filters. Experimental examinations of a large number of combinations of tapers to enlarged guides and corresponding abrupt steps to the same size guide produced a quantity of empirical data from which the following conclusions could be drawn.

1) The presence of the tapers contributed to the excitation of the higher-order modes which are necessary for the existence of the high insertion losses in the single section.

2) Of the modes capable of propagating in the enlarged waveguide (TE_{10}, TE_{11}, TE_{12}, TE_{01}, TE_{02}, TM_{11}, and TM_{12}), only the TE_{12} and TM_{12} modes can be considered as contributing to this resonant insertion loss.

3) Though the evidence is not conclusive, it is quite likely that it is the TE_{12} mode in which the high insertion loss elements operate.

4) The high insertion loss resonances show a relatively high Q (narrow bandwidth).

5) Discontinuities, or tuning in the standard waveguide on either the load or generator side of the filter section, do not affect the frequency of the high insertion loss and only produce minor changes in the value of the insertion loss itself.

6) The assumption of the TE_{12} mode in the enlarged waveguide permits the establishment of empirical design criteria leading to practical high insertion loss filters.

The Resonance Phenomena

A waveguide two-port structure is described in which each of the modes TE_{10}, TE_{12}, etc., form independent waveguide two-port structures coupled by ideal transformers at input and output ports to the main propagating mode line. One of the two-port structures carries the TE_{10} mode and can be treated as in Vogelman[1] except near the high insertion loss frequencies. When the TE_{12} mode becomes resonant, it extracts essentially all of the RF power from the fundamental mode-transmission line. For the TE_{12} mode, the enlarged waveguide section is terminated by a waveguide beyond cutoff at each end. Since neither the TE_{12} mode nor its related radial mode will propagate towards the standard waveguide for any distance, the energy coupled into this resonant cavity is dissipated in the metallic walls at resonance, producing a high insertion loss at that frequency. This concept was checked by the following experiment. A signal at resonance (*i.e.*, high insertion loss frequency) was fed through a slotted line in series with a directional coupler, a slide screw tuner, the filter, another directional coupler and a matched load. The magnitude of the insertion loss was measured with the slide screw tuner effectively out of the circuit. The VSWR was measured with the slotter line and the slide screw tuner adjusted to produce a VSWR of less than 1.05. The insertion loss was measured again. Without the tuner, the VSWR's that were measured were in the order of 5, or less, with insertion losses of >40 db. When the VSWR was reduced to less than 1.05, the insertion-loss measurement showed only a decrease of less than 1.5 db in all cases. This is the effect which is expected when a dissipative element is inserted in a transmission line. This experiment lends credence to the conclusion that the filter element at resonance in the TE_{12} mode is not reciprocal insofar as its mode-transformation properties are concerned; *i.e.*, it readily transforms TE_{10} power to the TE_{12} mode, but does not transform TE_{12} power back to the TE_{10} mode to any appreciable extent. In the case of multiple modes, the normal inductive or capacitive coupling of a resonant circuit to a transmission line cannot be used to describe the performance. A theoretical analysis of the equivalent circuit of the discontinuities between the enlarged uniform line and the radial line at each end and the reactance caused by the waveguide beyond cutoff radial lines was atttempted without success insofar as producing a relatively exact treatment of the high insertion loss phenomena was concerned. However, the analysis leads to the following qualitative conclusions.

1) The resonant length of the enlarged uniform line is reduced from one-half waveguide wavelength by the reactances of the discontinuities and the waveguide beyond cutoff at each end of the uniform line.

2) For short enlarged uniform lines, the interaction between the discontinuities at each end further shortens the required line length for resonance.

3) The section will resonate even when the line length of the enlarged uniform guide is reduced to zero because of the presence of the discontinuity reactances and the waveguides beyond cutoff.

Since an analytic solution was not possible, design formulas were empirically derived from the data experimentally obtained for enlarged waveguide sections.

General Design Considerations

A large number of filter sections, consisting of 45° tapers at the ends of an enlarged line, were constructed and tested to obtain empirical relations between the high insertion loss frequency and the dimensions of the cavity section. From the resultant data, the following empirical relationships were obtained.

1) The measured high insertion loss frequency occurs within one per cent of the value computed for a uniform enlarged waveguide length equal to a quarter wavelength for the TE_{12} mode, provided that this length is greater than one-half the cutoff wavelength.

2) When the enlarged waveguide length is reduced to zero, the high insertion resonance frequency is within three per cent, if the resonance is computed on the assumption that an equivalent length equal to the waveguide height is divided by $2\sqrt{2}$.

3) Linear interpolation can be used to compute the TE_{12} resonant wavelength for cavity length between zero and $\lambda_c/2$. This gives results which are within three per cent for lengths much shorter than $\lambda_c/2$ and closer to one per cent as the length approaches $\lambda_c/2$.

4) The Q of the TE_{12} resonance defined in terms of the bandwidth of the points 3 db down from the maximum insertion loss, is a function of the ratio of the TE_{12} resonant frequency to the cutoff frequency for that mode. Typical values of Q as determined in accordance with the preceding definition are as follows:

a) four to five thousand within one to two per cent of the cutoff frequency with a corresponding frequency-to-width ratio at 20-db insertion loss of approximately 200 to 300;

b) two thousand to twenty-five hundred within five to six per cent of the cutoff frequency with a corresponding frequency-to-width ratio at 20-db insertion loss of approximately 75 to 150;

c) four to five hundred for frequencies within twelve to thirteen per cent of the cutoff frequency with ratio at 20-db insertion loss of approximately 50 to 75;

d) two hundred and fifty to three hundred when the length of the enlarged line is reduced to zero, corresponding to a frequency-to-width ratio at 20-db insertion loss of 30 to 50;

e) the second-, third- and higher-order insertion-loss frequencies have correspondingly lower Qs.

Design Formulas

Fig. 1 shows the dimensions to be used in computing the resonant frequency of a high insertion loss filter section: l is the length of the enlarged section; a is the width of both the standard waveguide and the enlarged waveguide; b is the height of the standard waveguide, and b_2 is the height of the enlarged waveguide.

The notation λ_g and λ_c will refer to the waveguide wavelength and the cutoff wavelength of the enlarged uniform waveguide for the TE_{12} mode. The cutoff wavelength for the TE_{12} mode for a waveguide of width a and height b_2 is given by the formula[2]

$$\lambda_{c_{12}} = \frac{2a}{\sqrt{1 + (2a/b_2)^2}}. \tag{1}$$

Note: This mode cutoff occurs when the waveguide wavelength in the TE_{10} mode is equal to the height b_2.

The waveguide wavelength is obtained from the cutoff wavelength from the relationship

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (\lambda/\lambda_{cnm})^2}} \tag{2}$$

which, for the TE_{12} mode, reduced to

$$\lambda_g = \frac{1}{\sqrt{1/\lambda^2 - 1/4a^2 - 1/b_2^2}} \tag{3}$$

for l_1 larger than $(2n-1)\lambda_c/2$ where n is the number of the resonance counting the resonances from lowest to higher frequencies. The required resonant dimension can be obtained from the resonant wavelength by the following relationship:

$$l_1 = (2n - 1)\lambda_g/4 \tag{4}$$

which combines with (3) to give

$$l_1 = \frac{2n - 1}{\sqrt{1/\lambda^2 - 1/4a^2 - 1/b_2^2}}. \tag{5}$$

Conversely, the resonant free-space wavelength is given in terms of the length, width and height by the following relationship:

$$\lambda_l = \frac{1}{\sqrt{\left(\frac{2n-1}{4l_1}\right)^2 + 1/4a^2 + 1/b_2^2}}. \tag{6}$$

Substituting

$$l_1 = (2n - 1)\lambda_{c_{12}}/2 \tag{7}$$

in (6) we obtain the criteria for the validity of (6); *i.e.*, that the TE_{12} resonant wavelength λ is smaller than λ_{lc} where λ_{lc} is given by

$$\lambda_{lc} = \frac{0.894427}{\sqrt{1/4a^2 + 1/b_2^2}}. \tag{8}$$

[2] N. Marcuvitz, "Waveguide Handbook," Mass. Inst. Tech. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., p. 88; 1951.

The TE_{12} resonant wavelength for l_1 equal to zero is found to be

$$\lambda_{l_0} = \frac{1}{\sqrt{3/2b_2^2 + 1/4a_2^2}}. \tag{9}$$

If values of l_1 between zero and $\lambda_c/2$ are used, the resonance can be found by linear interpolation between the values obtained from (6) and (8) or from the relationship

$$\lambda_r = \sqrt{3}.2l_1\left(1 - \frac{\lambda_{l_0}}{\lambda_{lc}}\right) + \lambda_{l_0}. \tag{10}$$

For the second, third and subsequent resonances where the length of the enlarged section l_1 is less than $(2n-1)\lambda_c/2$ for the TE_{12} mode, the high insertion loss wavelength is obtained from

$$\lambda_r = \lambda_l + \left[1 - \frac{\lambda_l}{\lambda_{lc}}\right]\frac{\sqrt{3}.2l_1}{(2n - 1)}, \tag{11}$$

where λ_l is the value computed from (6). Eqs. (10) and (11) will give values of the high insertion loss wavelength to an accuracy of three per cent for the second, third and fourth TE_{12} resonance, provided that a/λ is greater than 1 or appropriate steps have been taken to suppress the TE_{20} mode in the standard-size waveguide.

Filter Design Procedure

A band-rejection filter using high insertion loss sections is specified in terms of the upper and lower cutoff frequencies of the pass band, the lowest high insertion loss frequency, and the minimum acceptable highest frequency of the attenuation band. The following symbols will be used.

- λ_a is the wavelength corresponding to the lower cutoff frequency of the pass band.
- λ_b is the wavelength corresponding to the upper cutoff frequency of the pass band.
- λ_{m_n} is the lowest frequency for the $(2n-1)\lambda_g/4$ resonance.
- λ_{M_n} is the highest frequency for the $(2n-1)\lambda_g/4$ resonance.

The procedure for designing a band-rejection filter consists of selecting a series of filter sections of the high insertion loss type such that the frequencies over which they reject form in combination a continuous frequency band of the desired width. The rejection bandwidth is divided into several parts, the lowest frequency part being covered by the filters operating in the first resonance, that is, in the $\lambda_g/4$ condition. The second band, which is continuous with the first, consists of the same filters operating in the $3\lambda_g/4$ mode and so forth. The first step in the design procedure is to select λ_{M_1}, the wavelength corresponding to the highest frequency of the rejection section in the $\lambda_g/4$ mode. If this value is selected to be equal in wavelength to that resulting from the application of (8), the simplest design procedure results. The criteria for selection of λ_{M_1} (derived in Appendix I) are

$$0.894427\lambda_{m_1} \leq \lambda_{M_1} \leq 0.906765\lambda_{m_1}, \tag{12}$$

where λ_{m_1} is wavelength of lowest attenuation frequency. Normally, λ_{M_1} is selected close to the upper limit in order to produce individual sections of maximum rejection bandwidth. By manipulation of (8), we obtain the relationship for determining the height of the enlarged waveguide to be used in the filter design.

$$b_2 = \frac{1}{\sqrt{\frac{0.8}{\lambda_{M_1}} - \frac{1}{4a^2}}} \cdot \tag{13}$$

The length of the section l_1 corresponding to the wavelength λ_{M_1} is found from (5) and (13) to be

$$l_{M_1} = \frac{\lambda_{M_1}}{\sqrt{3.2}} \cdot \tag{14}$$

The length of the enlarged waveguide in the filter section for the lowest attenuation frequency is given by the relationship

$$l_{m_1} = \frac{1}{4\sqrt{\frac{1}{\lambda_{m_1}^2} - \frac{0.8}{\lambda_{M_1}^2}}} \cdot \tag{15}$$

Selection of λ_{M_1}, in accordance with the criteria of (12), insures that the resonances of the first, second, third, etc., type will overlap and form a continuous attenuation band provided that sufficient sections of lengths between l_{M_1} and l_{m_1} are incorporated to insure adequate attenuation over the band from λ_{m_1} to λ_{M_1}. Between 10 and 24 sections have been found to be adequate for filling the band between these two extremes. The exact number is dependent on the ripple in the attenuation curve which can be tolerated. As a first approximation, linear interpolation of the lengths between the two extremes has been found to be completely adequate if an adequate overlap is provided to account for tolerances. Alternately, sections spaced 0.5 to 1 per cent apart can be separately computed between λ_{m_1} and λ_{M_1}. The spacing between the high insertion-loss sections is computed from TE_{10} mode characteristics of the sections by the methods given by Vogelman.[1] This consists of computing quarter waveguide wavelength sections at the center frequency of the pass band between the equivalent steps corresponding to each of the high insertion loss sections. The resultant reflection loss at λ_a and λ_b is computed by the same method. If the insertion losses at the extremes of the pass band are too high, lengths of different values can be used to decrease these values at the expense of an increased insertion loss at the center frequency.

Filter Performance

Filters designed by the method described have been constructed and measured. Fig. 2 is an example of the filter performance when too few sections are selected.

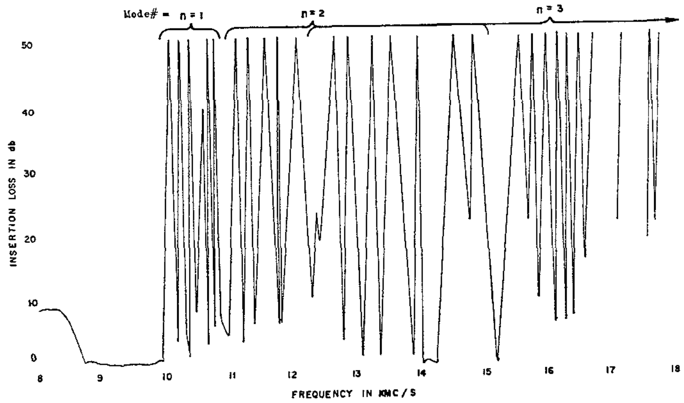


Fig. 2—Insertion loss performance of six-section filter.

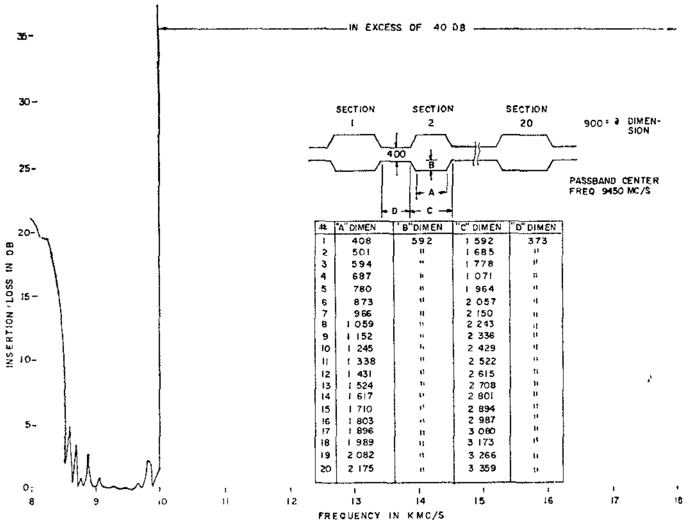
The original calculations were intended to cover the band from 10,000 to 11,000 mc in 0.900×0.400 ID waveguide. The differences can be directly attributed to the mechanical tolerances and the 1 per cent accuracy of the equations. The 20-db insertion loss bandwidth of any section is between 75 and 100 mc for the lowest resonance mode ($n=1$). Dissymmetries in a section (as seen for section 4) result in a reduction of the peak attenuation of the section. For the higher resonance modes the dissymmetries are more pronounced and show themselves as splittings of the absorption curves and extraneous resonance peaks. When seven more sections were added to this filter, resonant at intermediate frequencies in the band between 10,000 and 11,000 mc, the depth of the nulls in the lowest resonance mode was never less than 10 db, and for frequencies above 12,000 mc, never less than 25 db. When the total number of sections was increased to 20, the attenuation at all frequencies from 10,000 to beyond 18,000 mc was greater than 40 db (Figs. 3 and 4). This filter was subjected to high power, with and without pressurization in the pass band at 9450±50 mc with the following results.

No pressure: intermittent arcing at 350 kw.
15-pound pressure: no arcing at over 700 kw.

A single section was tested at its high insertion loss frequency at a power level of 50 kw. This is in excess of the reflected power that would cause magnetron breakdown. No breakdown in the filter occurred. Since this value is much higher than the spurious output of useable transmitter tubes, no further testing was considered necessary.

Conclusions

The procedure for designing high-power microwave rejection filters using higher-order modes has been described. These filters appear most promising as an aid in



#	"A" DIMEN	"B" DIMEN	"C" DIMEN	"D" DIMEN
1	408	592	1 592	373
2	501	"	1 685	"
3	594	"	1 778	"
4	687	"	1 071	"
5	780	"	1 964	"
6	873	"	2 057	"
7	966	"	2 150	"
8	1 059	"	2 243	"
9	1 152	"	2 336	"
10	1 245	"	2 429	"
11	1 338	"	2 522	"
12	1 431	"	2 615	"
13	1 524	"	2 708	"
14	1 617	"	2 801	"
15	1 710	"	2 894	"
16	1 803	"	2 987	"
17	1 896	"	3 080	"
18	1 989	"	3 173	"
19	2 082	"	3 266	"
20	2 175	"	3 359	"

Fig. 3—Dimension and insertion loss performance of 20-section filter.

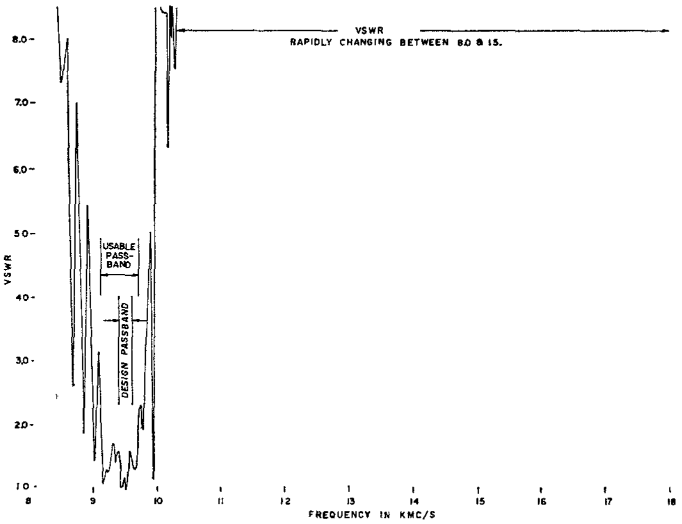


Fig. 4—VSWR performance of 20-section filter.

solving the major problems of radar-to-radar and radar-to-communications interference. In particular, the high insertion loss elements offer great promise in reducing the narrow-band spurious responses from radar magnetrons, since the elements can easily be cascaded to give insertion losses at particular frequencies of over 100 db and since the addition of matching structures for the pass band has no effect on the high insertion loss element. It has been found that cascading of filters designed in the manner indicated can give insertion losses as high as 120 db over a bandwidth of 40 per cent of the operating frequency. The power capability at frequencies in the pass band has proven to be equal to that encountered in normal waveguides with the usual quantity of joints and bends. This type of filter, though designed specifically for the high-power use, will find application in the low-power field where high insertion loss is required.

Appendix I

For a real solution, the value under the radical of (15) establishes the requirement that

$$\lambda_{M_1} \geq \lambda_{m_1}\sqrt{0.8}. \quad (16)$$

For the general case, for a continuous rejection band without holes

$$\lambda_{m_n} \geq \lambda_{M_{n-1}}; \qquad n \geq 2. \quad (17)$$

From (6) and (15),

$$\lambda_{m_n} = \frac{1}{\sqrt{\left(\frac{2n-1)^2}{4l_{m_1}}\right)^2 + \frac{0.8}{\lambda_{M_1}{}^2}}}$$

$$= \frac{1}{\sqrt{\left(\frac{2n-1}{\lambda_{m_1}{}^2}\right)^2 - 0.8(2n-1)^2 - 0.8}}. \quad (18)$$

From (6) and (14),

$$\lambda_{M_{n-1}} = \frac{1}{\sqrt{\left(\frac{2n-3}{4l_{M_1}}\right)^2 + \frac{0.8}{\lambda_{M_1}{}^2}}}$$

$$= \frac{\lambda_{M_1}}{\sqrt{0.2(2n-3)^2 + 0.8}}. \quad (19)$$

If (16) is to hold, then substituting (18) and (19) in (16),

$$\frac{1}{\frac{(2n-1)^2}{\lambda_{m_1}{}^2} - \frac{0.8(2n-1)^2 - 0.8}{\lambda_{M_1}{}^2}} \geq \frac{\lambda_{M_1}{}^2}{(0.2(2n-3)^2 + 0.8)}, \quad (20)$$

$$\frac{0.2(2n-3)^2 + 0.8(2n-1)^2}{(2n-1)^2} \lambda_{m_1} \geq \lambda_{M_1}, \quad (21)$$

$$\lambda_{M_n} \leq \sqrt{0.2\left(\frac{2n-3}{2n-1}\right)^2 + 0.8}\, \lambda_{m_1}; \quad n \geq 2. \quad (22)$$

The maximum restriction occurs when $2n-3/2n-1$ is a minimum; *i.e.*, when $n=2$.

Thus

$$\lambda_{M_1} \leq \sqrt{0.8222 \cdots}\, \lambda_{m_1} \quad (23)$$

which proves (12).